Power Control for Wireless Interference Network

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I. Abstract

A sum rate optimization for a wireless network has been considered in this problem. In this wireless network, each link will receive not only its desired signal but also interference from other links. Since the original optimization problem is not a convex optimization problem, we propose several methods to transform the original objective function into a concave form. Then, dual-based sub-gradient methods have been proposed to deal with each transformations. The simulation results show that our proposed algorithm can converge in an acceptable number of iterations. Besides, the comparison between each proposed methods is discussed via simulation.

II. Problem Introduction and Statement

In this problem, we consider a wireless network with n data links. Each link i transmits with power P_i. Since the transmission antenna in each link is omnidirectional, its transmission signal will not only transmit to its target receiver but also interfere to other links. Therefore, the influence of interference can not be ignored in our considered model. Signal-to-interference-plus-noise ratio (SINR) is a common metric in wireless communication to indicate the quality of wireless connections. SINR is considered as the power of desired signal divide by the received interference and noise, which makes us formulate the SINR of link i as

$$\gamma_i = \frac{G_{i,i}P_i}{\sigma_i^2 + \sum_{j\neq i} G_{i,j}P_j}, \tag{1}$$

in (1), $G_{i,i}$ is the channel gain between the transmitter and receiver on link i and $G_{i,j}$ is the channel gain between the transmitter on link i and the receiver on link j. σ_i^2 is the thermal noise. Hence, the received interference of the receiver on link i can be represented as $\sum_{j\neq i} G_{i,j}P_j$.

According to the Shannon-Hartley theorem, the data rate of link i can be expressed as

$$R_i = W\log_2(1+\gamma_i), \tag{2}$$

where W is the bandwidth of each link. However, since the constant bandwidth W and changing $\log_2$ to log have no effect to the solution of the optimization problem, the data rate of this problem has been modified into

$$R_i = \log(1+\gamma_i). \tag{3}$$

This problem aims to find appropriate power allocation to maximize the total utility. However, for maximizing the utility function, the adjusted power allocation is inside two log function, which makes this problem hard to solve. Based on the professor's suggestions, we change the original optimization problem to total sum rate optimization problem. In addition, by comparing the original utility optimization problem and total sum rate optimization problem, we propose an additional complicated optimization problem which also considers minimum data rate requirement. On the other hand, the objective function contains the interference part, which is a non-concave function. Inspired by [1] and [2], we provide two ways to transform the original non-concave optimization problem into concave problem. Furthermore, dual-based sub-gradient method has been proposed to solve the transformed optimization problem. Difference from directly using gradient method that can only find local optimal value due to the non-convex form, our transformation and proposed method can guarantee to find optimal value in an acceptable iteration's range. The detailed explanation will be introduced in the following section.

III. Problem Formulation and Solutions

Our goal is to adjust the transmission power to maximize the total sum rate of the considered network, while ensuring the allocated transmission power will not exceed its maximal power supply. The optimization problem can be presented as follow:

$$\max_{\mathbf{P}} \quad \sum_{i=1}^{N} R_i \tag{4a}$$

$$\text{s.t. } 0 \leq P_i \leq P_i^{max}, \forall i, \tag{4b}$$

where $\mathbf{P} = \{P_i | \forall i\}$ and (4b) is the power allocation constraint. However, the objective function in (4) has a difference of convex (d.c) structure, which is nonconcave.
Proof: we can rewrite the objective function in (4) as follow

$$\sum_{i=1}^{N} R_i = \sum_{i=1}^{N} [\log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} P_j + G_{i,i} P_i) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} P_j)], \tag{5}$$

(5) is a d.c structure. According to [3], a function that is a d.c structure is not a concave function. Thus, further transformation is needed to get the optimal solution.

A. *High SINR approximation*

Firstly, we assume high SINR case, i.e., $\gamma_i \gg 1$. In this case, the data rate of link i can be approximated to $\widehat{R}_i = \log(\gamma_i)$. Then, using the technique of geometric programming, we define $\widehat{P}_i = \log P_i$. As such, the original objective function can be transformed into

$$\sum_{i=1}^{N} \widehat{R}_i = \sum_{i=1}^{N} [\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} \exp(\widehat{P}_j))], \tag{6}$$

since (6) is a log-sum-exp function, which is a concave function [2], the original optimization problem can be reformulated as

$$\max_{\widehat{\mathbf{P}}} \quad \sum_{i=1}^{N} [\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} \exp(\widehat{P}_j))] \tag{7a}$$

$$\text{s.t. } \exp(\widehat{P}_i) \leq P_i^{max}, \forall i, \tag{7b}$$

By such transformation, the optimization problem has been transformed into concave maximization problem. Therefore, Lagrange dual technique [4] can be used to deal with this problem.

Lagrange function can be formulated as

$$L(\widehat{\mathbf{P}}, \boldsymbol{\lambda}) = \sum_{i=1}^{N} [\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} \exp(\widehat{P}_j))] - \sum_{i=1}^{N} \lambda_i [\exp(\widehat{P}_i - P_i^{max})], \tag{8}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector corresponding to power allocation constraint. The dual problem then can be written as

$$\min_{\boldsymbol{\lambda}} \max_{\widehat{\mathbf{P}}} L(\widehat{\mathbf{P}}, \boldsymbol{\lambda}). \tag{9}$$

The dual problem can be solved by decomposing it into two nested loops: to maximize $\widehat{\mathbf{P}}$ for given $\boldsymbol{\lambda}$ in the outer loop and to minimize the dual problem through $\boldsymbol{\lambda}$ in the outer loop. Therefore, by differentiating $L(\widehat{\mathbf{P}}, \boldsymbol{\lambda})$ with respect to $\widehat{P}_i$ and replacing $\widehat{P}_i = \log(P_i)$, the corresponding KKT condition can be written as

$$1 - \lambda_i P_i - P_i \sum_{j \neq i} \frac{G_{j,i}}{\sum_{k \neq j} G_{k,j} P_k + \sigma_j^2} = 0, \tag{10}$$

by arranging (10) in terms of P_i, the optimal power allocation in each iteration can be obtained

$$P_i = \frac{1}{\lambda_i + \sum_{j \neq i} \frac{G_{j,i}}{\sum_{k \neq j} G_{k,j} P_k + \sigma_j^2}}. \tag{11}$$

Besides, we use subgradient method to solve the dual problem, i.e. $\min_{\boldsymbol{\lambda} > 0} L(\widehat{\mathbf{P}}, \boldsymbol{\lambda})$, in the outer loop. The Lagrange multiplier update function can be written as

$$\lambda_i(k+1) = [\lambda_i(k) - \zeta(k)(P_i^{max} - P_i)]^+, \tag{12}$$

where $[x]^+ = \max\{0, x\}$, k is the iteration index, $\zeta(k)$ is the step size, which we set as 1 in the simulation in order to get the optimal result.

B. High SINR approximation with minimum data rate constraint

Compare the difference between log utility objective function and sum rate objective function, due to the property of log function, log utility objective function will guarantee minimum data rate of each link by avoid allocating power unduly toward certain link. However, to maximize the sum rate objective function, power might only allocate to certain link to mitigate the influence of interference, which cause other links lost the ability of data transmission.

For the reason mentioned above, we reformulate the optimization problem in III-A with additional minimum data rate requirement.

$$\max_{\mathbf{P}} \quad \sum_{i=1}^{N} R_i \tag{13a}$$

$$\text{s.t. } 0 \leq P_i \leq P_i^{max}, \forall i \tag{13b}$$

$$R_i \geq R_{min}, \forall i, \tag{13c}$$

where R_{min} is the minimum data requirement. By adopting same manner of geometric programming, the transformed optimization problem can be rewritten as

$$\max_{\widehat{\mathbf{P}}} \quad \sum_{i=1}^{N} [\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} \exp(\widehat{P}_j))] \tag{14a}$$

$$\text{s.t. } \exp(\widehat{P}_i) \leq P_i^{max}, \forall i \tag{14b}$$

$$R_i \geq R_{min}, \forall i. \tag{14c}$$

Then, the corresponding Lagrange function is formulated as

$$\begin{aligned} L(\widehat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \sum_{i=1}^{N} [\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j \neq i} G_{i,j} \exp(\widehat{P}_j))] \\ & - \sum_{i=1}^{N} \lambda_i [\exp(\widehat{P}_i) - P_i^{max}] + \sum_{i=1}^{N} \mu_i [\widehat{R}_i - R_{min}], \end{aligned} \tag{15}$$

where $\boldsymbol{\mu}$ is the Lagrange multiplier vector corresponding to minimum data rate requirement. The optimal power allocation in inner loop is calculated as

$$P_i = \frac{1+\mu_i}{\lambda_i + \sum_{j\neq i}(1+\mu_j)\frac{G_{j,i}}{\sum_{k\neq j} G_{k,j}P_k+\sigma_j^2}}, \tag{16}$$

the Lagrange multiplier update functions are written as

$$\lambda_i(k+1) = [\lambda_i(k) - \zeta_1(k)(P_i^{max} - P_i)]^+, \tag{17}$$

$$\mu_i(k+1) = [\mu_i(k) - \zeta_2(k)(\widehat{R}_i - R_{min})]^+. \tag{18}$$

C. Lower bound approximation with minimum data rate constraint

The above subsections consider the case of high SINR, however, this case is not necessarily held in general. Thus, a lower bound approximation is provided to solve the problem.

The lower bound of data rate can be expressed as [2]

$$R_i = \log(1+\gamma_i) \geq \alpha_i \log(\gamma_i) + \beta_i, \tag{19}$$

where α_i and β_i can be updated as

$$\alpha_i = \frac{\gamma_i}{1+\gamma_i}, \tag{20}$$

$$\beta_i = \log(1+\gamma_i) - \frac{\gamma_i}{1+\gamma_i}\log(\gamma_i). \tag{21}$$

We then reformulate the transformed optimization problem as

$$\max_{\widehat{\mathbf{P}}} \quad \sum_{i=1}^{N} \alpha_i[\widehat{P}_i + \log(G_{i,i}) - \log\left(\sigma_i^2 + \sum_{j\neq i} G_{i,j}\exp(\widehat{P}_j)\right)] + \sum_{i=1}^{N}\beta_i \tag{22a}$$

$$\text{s.t. C1: } \exp(\widehat{P}_i) \leq P_i^{max}, \forall i, \tag{22b}$$

$$\text{C2: } R_i \geq R_{min}, \forall i, \tag{22c}$$

$$\tag{22d}$$

the corresponding Lagrange function is written as

$$L(\widehat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{N} \alpha_i[\widehat{P}_i + \log(G_{i,i}) - \log(\sigma_i^2 + \sum_{j\neq i} G_{i,j}\exp(\widehat{P}_j))] + \sum_{i=1}^{N}\beta_i \tag{23}$$

$$- \sum_{i=1}^{N}\lambda_i[\exp(\widehat{P}_i - P_i^{max})] + \sum_{i=1}^{N}\mu_i[\widehat{R}_i - R_{min}]. \tag{24}$$

Furthermore, the optimal power allocation in each iteration is calculated as

$$P_i = \frac{\alpha_i+\mu_i}{\lambda_i + \sum_{j\neq i}(\alpha_j+\mu_j)\frac{G_{j,i}}{\sum_{k\neq j} G_{k,j}P_k+\sigma_k^2}}. \tag{25}$$

Using subgradient method to update the Lagrange multiplier, the update functions are written as

$$\gamma_i(k+1) = [\gamma_i(k) - \zeta_1(k)(P_i^{max} - P_i)]^+, \tag{26}$$

$$\mu_i(k+1) = [\mu_i(k) - \zeta_2(k)(\widehat{R}_i - R_{min})]^+. \tag{27}$$

Note that we use lower bound to approximate the data rate, thus, we use (20) (21) to update the lower bound until the lower bound converges to the original data rate. The pseudo code of the proposed method is shown in **Algorithm 1**.

Algorithm 1: Proposed Algorithm

Initialize the maximum number of iterations in C_o^{max}
Set the iteration index of lower bound update loop $c_o = 0$
repeat {Lower bound update Loop}
  Initialize the coefficients α_i and β_i in (19)
  Initialize the maximum number of iterations in dual problem loop C_i^{max}
  Set the iteration index of dual problem loop $k = 0$
  repeat {Dual Problem Loop}
    Solve the problem in (22)and obtain the decision policy $\mathbf{P}$ by (25)
    Update Lagrange multipliers by (26) (27)
    $k = k + 1$
  until Convergence of decision policy $\mathbf{P}$ or $k = C_i^{max}$
  Update α_i and β_i using equation (20) (21)
until Convergence of α_i and β_i or $c_o = C_o^{max}$

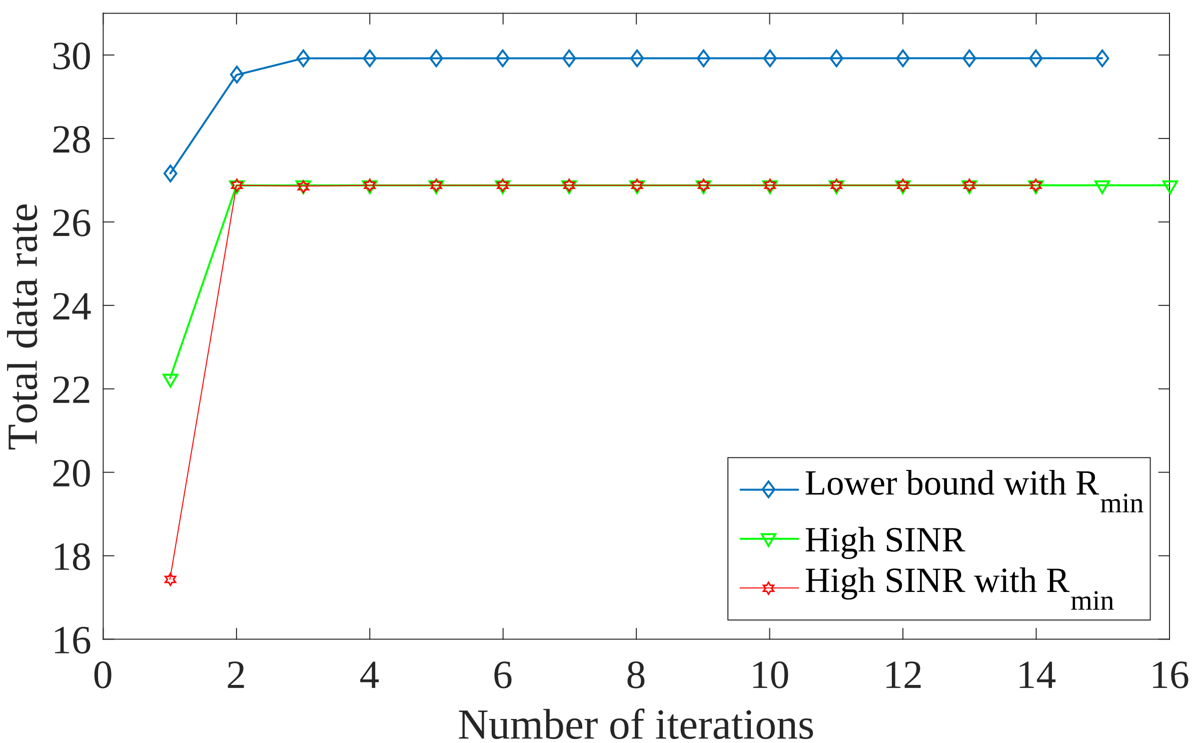


Fig. 1. Total data rate versus number of iterations in good channel condition.

IV. Simulation results

In this section, the performance of proposed algorithms are evaluated via simulations. Consider a wireless network consisting of 20 data links. The maximum transmission power of each link P_i^{max} is 0.2 Watt. The thermal noise σ_i^2 is -120 dBm. In order to guarantee non-negative data rate, the minimum data requirement R_{min} is 0. Convergence stopping criterion is set as 10^{-5}.

In Fig. 1, total data rate comparison between three different methods are provided over the number of iterations in good channel condition. In this case, good channel condition means $G_{i,i} \gg G_{i,j}$, i.e. interference has less influence on SINR. The results of "High SINR", "High SINR with R_{min}", and "Lower bound with R_{min}" are the proposed methods mentioned in section III-A, III-B, and III-C

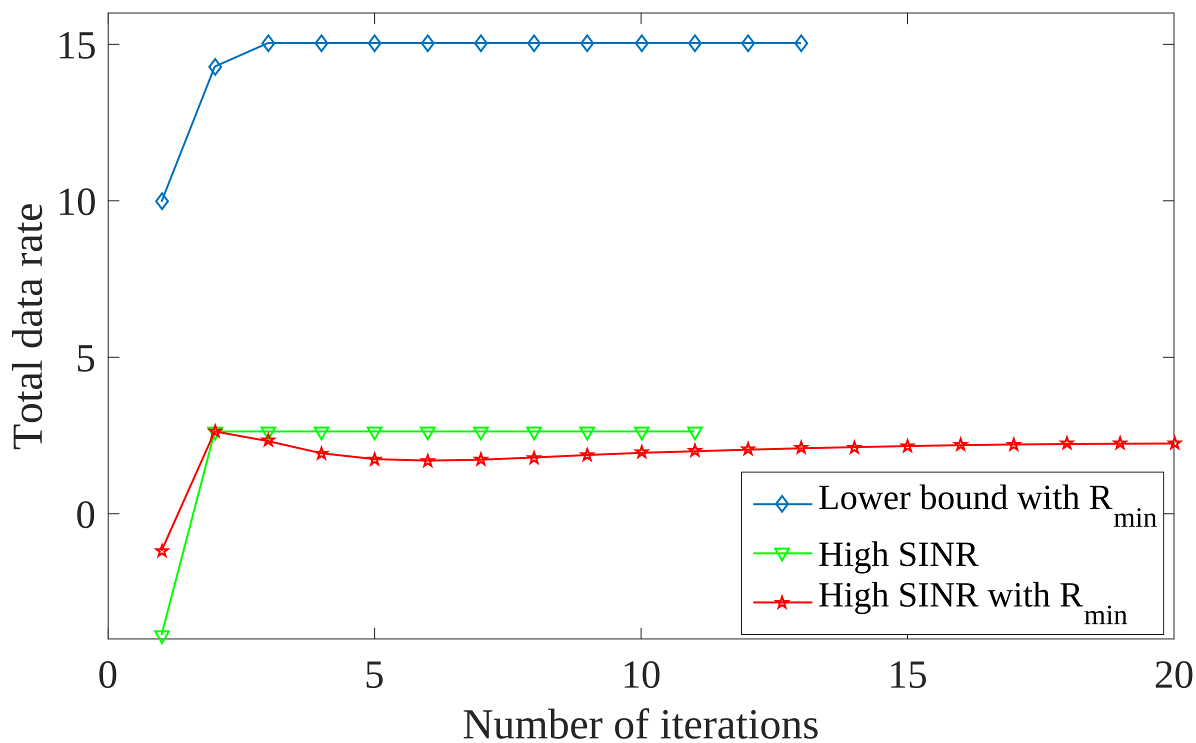


Fig. 2. Total data rate versus number of iterations in bad channel condition.

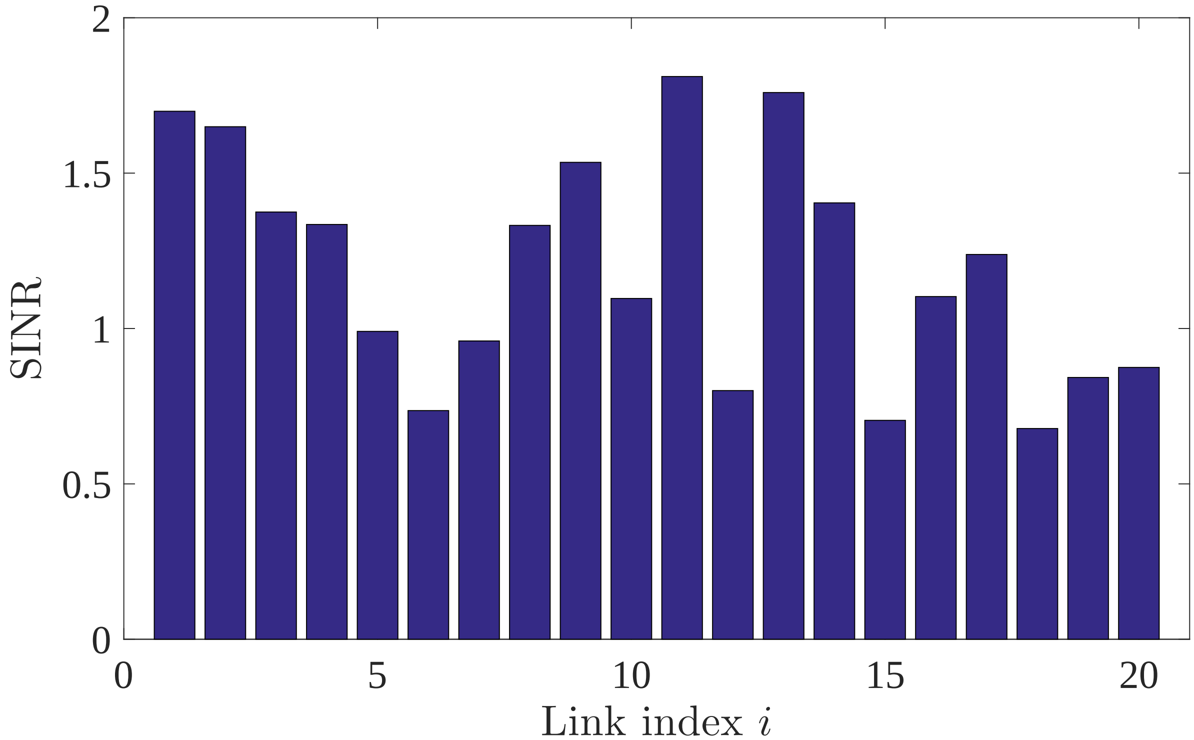
individually. It can be observed, firstly, that the proposed methods can converge in at most 16 times iterations. Additionally, the total data rate of "High SINR" case and "High SINR with minimum data rate constraint" case converge to same value since every links can satisfy minimum data requirement without specific constraint in good channel condition. Furthermore, since we update the lower bound until it converges to the original data rate, the lower bound approximation can get more accurate value of data rate than High SINR assumption. Therefore, "Lower bound with R_{min}" can get higher total data rate compared to other two High SINR cases.

In Fig. 2, we compare total data rate of proposed methods in bad channel condition, in which interference will have severe impact on SINR. In this case, power will allocate exceedingly toward certain link to enhance its data rate while sacrificing other links capability to reduce the influence of interference. Therefore, "Higher SINR" can get higher total data rate than "High SINR with R_{min}" since the latter one will guarantee minimum data rate of each link while sacrificing total throughput. Further SINR analyses will be introduced in Fig.3. On the other hand, "Lower bound with R_{min}" still get the highest value. It is because the assumption of $\gamma_i \gg 1$ is not held in bad channel condition.

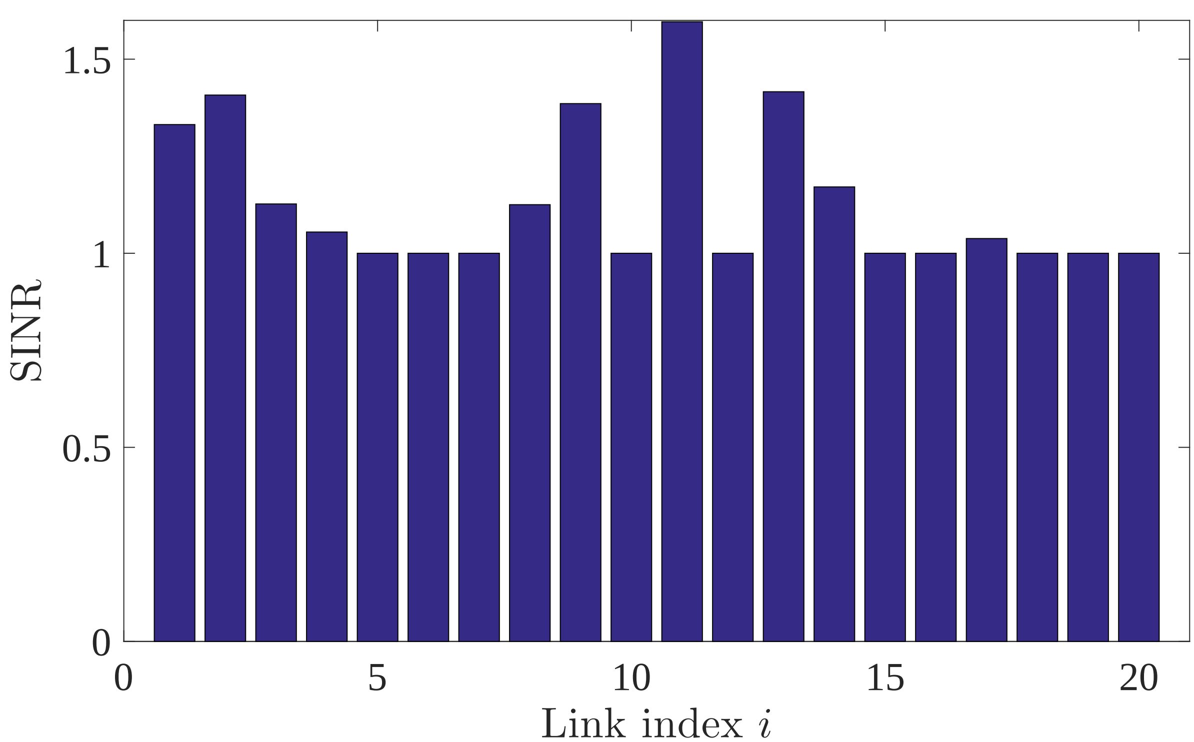
The results of SINR in each link using high SINR assumption method are illustrated in Fig.3. Fig.3(a) is the SINR without minimum data rate constraint whereas SINR with minimum data rate requirement is depicted in Fig.3(b). Since we restrict the minimum data rate R_{min} to 0, i.e, the minimum SINR is 1, it can be observed that some of links have been sacrificed in order to maximize the total data rate if minimum data rate constraint is not considered. However, if minimum data rate requirement is guaranteed, each link can fulfill its minimum SINR and yet its total data rate is lower than the case without minimum data rate constraint.

V. Conclusion and future work

In this problem, a sum rate optimization problem has been considered. Since the original optimization problem has a difference of convex structure, which is not a concave function, we consider



(a) Without minimum data rate constraint



(b) With minimum data rate constraint

Fig. 3. SINR of each data link

both high SINR approximation and lower bound approximation to transform the original problem into a concave form. In addition, by comparing the difference between utility function and total sum rate function, we consider the optimization problem with additional minimum data rate constraint. Furthermore, dual-based sub-gradient method has been proposed to deal with these three different optimization problems. Simulation results then compare the results of these three different cases in different channel condition. The simulation results show that lower bound approximation can get the highest value compare to others. The case with considering minimum data rate constraint will get the same value as the case without considering the additional constraint in good channel condition. However, in bad channel condition, the case without considering the additional constraint will get higher value than the case with considering minimum data rate constraint since the former case guarantees the minimum data rate. A distributed system network can be considered in the future work. Each link will not get the information of other links in the distributed network. Game theory optimization technique can be utilized to deal with the problem of distributed network.

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